Primary Mathematics Challenge – November 2018

Answers and Notes

These notes provide a brief look at how the problems can be solved.
There are sometimes many ways of approaching problems, and not all can be given here.
Suggestions for further work based on some of these problems are also provided.

P1 **E** 55 P2 **A** $\frac{1}{3}$

1 **D** 24 The number of balloons is $3 \times 4 + 4 \times 3 = 24$.

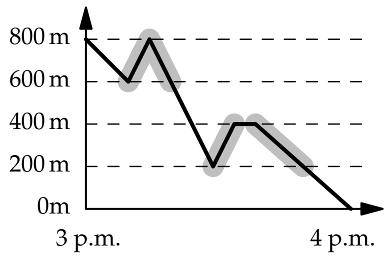
2 **D** 400 g We can convert 0.4 kg to 400 g.

3 **A** **ONION** The only letters of the alphabet with rotational symmetry are **H**, **I**, **N**, **O**, **S**, **X** and **Z**.

4 **D** 990 The number of years from William's birth is $2018 - 1028 = 990$.

5 **B** a page width We can convert 201.8 mm to a more convenient length, viz. 20.18 cm. Of the options the length of a big toe is nearer to 2 cm; the height of a door 200 cm; the distance across a sports stadium, 200 m; and the width of even a small galaxy like our puny Milky Way is around 100 000 light years or some 90 000 000 000 000 000 000 (or 90 *quintillion*) cm. The width of a page of your question paper is about 21 cm.

6 **E** 1600 m We can see that rather than just walking 800 m from his grandfather's house back home, Pat walked two 200 m sections twice more than he needed to (to take the cat back and to go back to read). So the total distance that he walked is $800 + 2 \times 2 \times 200 = 1600$ m.



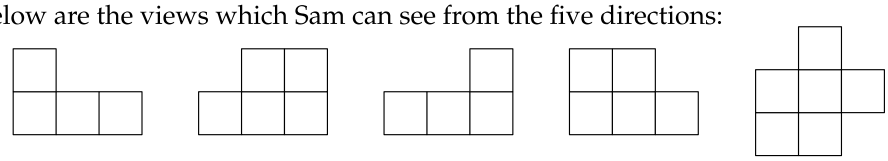
7 **E** 9.36 p.m. The number of minutes from the start of the match until the end is $45 + 3 + 15 + 45 + 3$, a total of 111 minutes. This is 1 hour and 51 minutes (or 9 minutes less than 2 hours), and so the match finished at 9.36 p.m.

8 **C** $\times$ not $-$ Since $0.4 = \frac{2}{5}$, multiplying 50.45 by 0.4 will calculate $\frac{2}{5}$ of 50.45, which is 20.18. The other options give very different answers: $50.45 + 0.4 = 50.85$, $50.45 - 40 = 10.45$, $50.45 - 04 = 46.45$ or $5045 - 0.4 = 5044.6$, and $50.45 \div 0.4 = 126.125$

9 **A** 30 Each of five positive even numbers is 1 larger than the corresponding odd number in 1, 3, 5, 7 and 9, which we are told have a sum of 25. So the sum of the first five positive even numbers is $25 + 5 = 30$.

10 **D** 81 Mr Fussy has 100 houses to choose from, but he will not choose the following: 4, 14, 24, 34, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 54, 64, 74, 84 or 94. Thus the number of houses he can choose is $100 - 19 = 81$.

11 **C** from right Below are the views which Sam can see from the five directions:



from left from front from right from back from above

12 **B** 20% The fraction of people living in Wales who speak Welsh is $\frac{600\,000}{3\,000\,000}$. This can be simplified to $\frac{1}{5}$, and so the percentage is 20%.

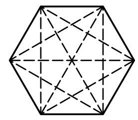
13 **C** £35 Since the amount that Mo had stolen was a multiple of 13 and also entirely in £5, it must be a multiple of £$13 \times 5 =$ £65. Hence Jo stole £$(100 - 65) =$ £35.

14 **E** 96 The number of fish fingers made from one block is $4 \times 6 \times 4 = 96$.

15 **B** 108 cm^2 The area of each square bite is $3 \times 3 = 9$ cm^2. The area of a piece of toast is $12 \times 12 = 144$ cm^2. So the area remaining after 4 bites is $144 - 4 \times 9 = 108$ cm^2.

16 **B** £1.25 Given that Teddy can get a cup of coffee and a cake for £3.75 while Freddy buys a cup of coffee and 2 cakes for £6.25, it must be that the cost of the cake is £6.25 − £3.75 = £2.50. So the cost of a cup of coffee is £3.75 − £2.50 = £1.25.

17 **B** 9 The diagonals of a hexagon are shown here – there are 9 of them (three from each of the six vertices).



18 **E** 12 December If we refer to 12 November as DAY 0, 13 November as DAY 1, 14 November as DAY 2 and so on, Mum changes her socks every day, Dad on each even-numbered day, Tammy on every day that is a multiple of 3; Timmy changes his every Wednesday, hence on every DAY that is 2 more than a multiple of 7. The first DAY that is all of these is DAY 30, which is 30 days after 12 November. Since November has 30 days, this is 12 December.

19 **D** 270° Each pair of angles at the three points where the square touches the triangle form a straight line with a right angle of the square between them. So each pair has a sum of $180° - 90° = 90°$. Thus the six shaded angles have a total of $3 \times 90° = 270°$.

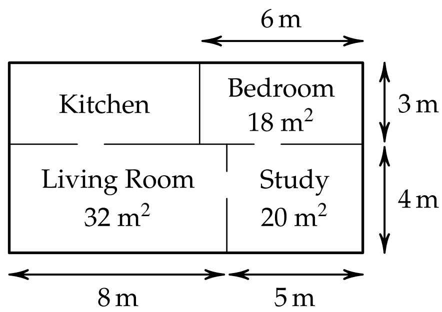
20 **C** 5 days We can make a table of Paige's reading progress day by day through her 444 pages:

	morning		evening	
day	pages read	pages left	pages read	pages left
1	20	424	212	212
2	20	192	96	96
3	20	76	38	38
4	20	18	9	9
5	9	finished		

So Paige will take 5 days to finish the book.

21 25 Anna thinks of a number, divides it by 5, adds to 20 to it and ends up with the number she first thought of. Let Anna's number be x. Then $x = \frac{x}{5} + 20$. Thus $x - \frac{x}{5} = \frac{4x}{5} = 20$ and so $x = 20 \div \frac{4}{5} = 25$.

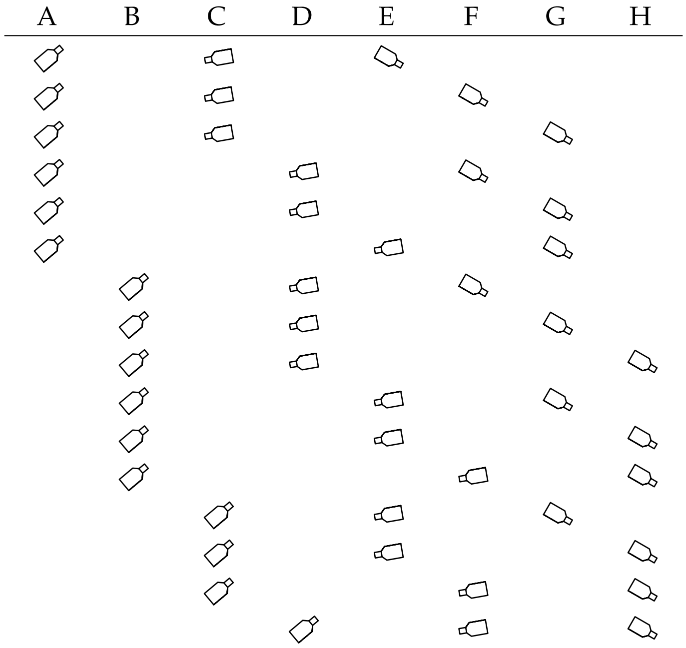
22 $21\,\text{m}^2$ Since the bedroom is twice as long as it is wide, and its area is $18\,\text{m}^2$, its length is 6 m and its width 3 m. Similarly for the living room, whose area is $32\,\text{m}^2$, its length and width are 8 m and 4 m respectively. For the study which shares the 4 m wall with the living room, and which has an area of $20\,\text{m}^2$, its length is 5 m. We can see that the kitchen measures $(8 + 5 - 6) = 7$ m by 3 m. So the area of the kitchen $= 7 \times 3 = 21\,\text{m}^2$.



23 897798 A six-digit palindrome looks like ABCCBA, where A, B and C are single digits. To be a multiple of 6, the last digit, A, must be even. To find the largest palindrome, we will start by assuming A = 8. Again to make the palindrome as large as possible, we shall also try B = 9. Any multiple of 6 is also a multiple of 3, and so the sum of the digits must have a total that is a multiple of 3. Hence A + B + C + C + B + A = 8 + 9 + C + C + 9 + 8 = 34 + 2C is a multiple of 3. Thus C could be 1, 4 or 7, and, choosing C to be 7, the largest palindrome is 897798.

24 94 Thinking of the problem in terms of ratios, and letting c, b and l represent of the number of cars, bikes and lorries respectively, we have $c : b = 1\frac{1}{3} : 1$, and $b : l = 1\frac{1}{4} : 1$. Simplifying both of these gives $c : b = 4 : 3$, and $b : l = 5 : 4$. Since we have some information about the total of c, b and l, it will useful to combine these two ratios; the number of bikes, b, is common to them both, so we can multiply the first by 5 and the second by 3. This leads to $c : b = 20 : 15$, and $b : l = 15 : 12$, whence $c : b : l = 20 : 15 : 12$. Hence the total number of vehicles is a multiple of $20 + 15 + 12 = 47$; being even and less than 100, the total must be $47 \times 2 = 94$.

25 16 We can label the eight friends with the letters A, B, C, D, E, F, G and H, and proceed carefully to enumerate the possibilities, noting that A and H cannot *both* have a popper:



The notes below suggest another approach to this problem.

Some notes and possibilities for further problems

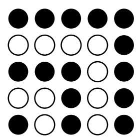
3 Can pupils find other words using only the letters **H**, **I**, **N**, **O**, **S**, **X** or **Z**.

4 This year is 2018. William the Conqueror was born in 1028. What other years have there been that use each of the digits 2, 0, 1 and 8 once and once only, and how many will there be similarly in the future?

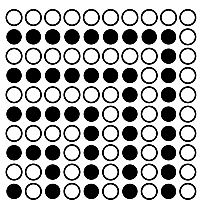
6 Pupils could find different story graphs (maybe even with a Dr Who-style time-warp where the lines would go from right to left).

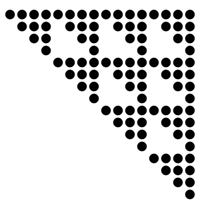
8 Assuming that Michaela correctly entered the numbers 5.045 and 0.4, how would she obtain answers of 4.645, or 5.445, or 12.1625, or even 1.910 488 . . . ?
Pupils might investigate whether there are many types of calculation where changing the operation does not affect the answer: e.g. $1 - \frac{1}{2} = 1 \times \frac{1}{2}$, or $0 \times 7 = 0 \div 7$, or $2^4 = 4^2$, or $2 + (\sqrt{3} - 1) = 2 \div (\sqrt{3} - 1)$.

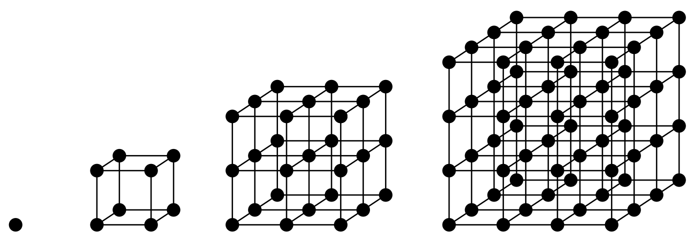
9 We can illustrate why it is that 25 (or any *square* number) is the sum of *consecutive odd numbers*. In the diagram on the right 25 is the sum of number of circles in each of the arrow-shaped bands: $1 + 3 + 5 + 7 + 9$.



The larger square number 100 is an interesting number for several other reasons. Because $10 = 1 + 2 + 3 + 4$ and because $100 = 10^2$, 100 is the square of a *triangular* number, and so it can also be represented in the fractal-like shape below, but also be written as the sum of consecutive cube numbers.







1 8 27 64

Also, the first 3 prime numbers (2, 3 and 5) add up to 10, but how many consecutive prime numbers, starting at 2, would you need to make a total of 100 ?
And also, 100 is the sum of consecutive integers in several different ways: $100 = 18 + 19 + 20 + 21 + 22$ or ${}^{-}8 + {}^{-}7 + {}^{-}6 + \ldots + 14 + 15 + 16$. Can you find the other ways?

There are several ways to write 100 using 9 digits:

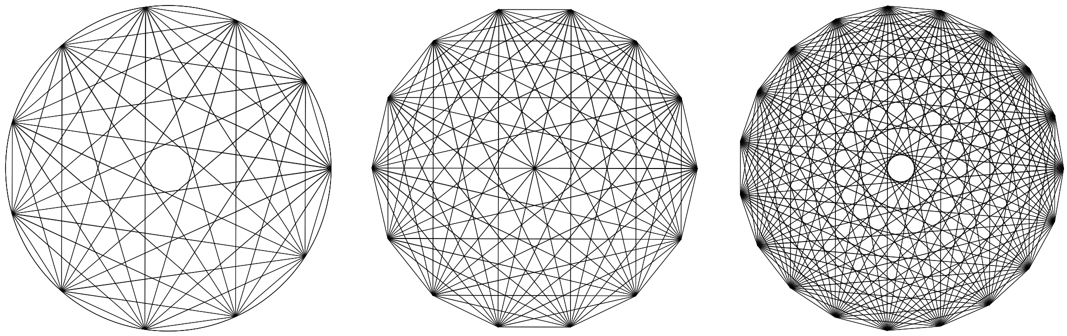
$$100 = 1+2+3+4+5+6+7+8\times 9 = 123+4-5+67-89 = 98-76+54+3+21.$$

And finally, centipedes do *not* have 100 legs: in fact all centipedes discovered so far have twice an odd number of legs, two for each body segment. The Common centipede has only 30 legs, but there is another species with as many as 394.

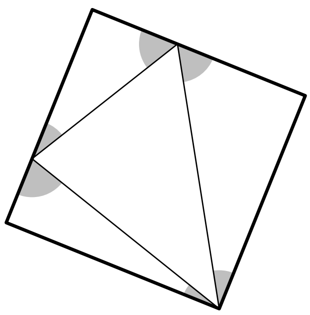
10 How many houses would the Fussys be able to consider if Mrs Fussy also decided that she wanted to avoid house numbers that include the digit 7? And what if Miss Fussy too declared that she does not want to be next door to a house whose numbers include a digit 0?

13 There are various ways of telling if a number is a multiple of 13. For large numbers, split the number into groups of three digits from the right, and then alternately subtract and add. For example: the number 29 407 638 gives $29 - 407 + 638 = 260$. If this answer is a multiple of 13, then the original number is also — as 260 is clearly 13×20, the original number 29 407 638 is also a multiple of 13.

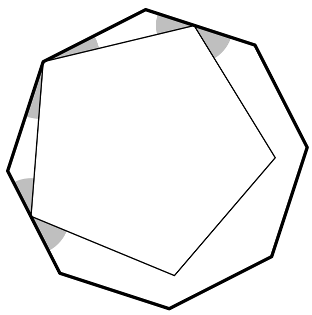
17 For each vertex, there are three points to which one cannot draw a diagonal: its two neighbours and the point itself. For an n-sided polygon, this would appear to lead to a total of $n \times (n-3)$ diagonals, but that would be to count each diagonal twice (once from each end) — so we must halve this. Therefore, for an n-agon, the number of diagonals is $\frac{1}{2}n(n-3)$. A diagram with all the diagonals and outside edges included is traditionally referred to as a *mystic rose*. Here are roses with 11, 14 and 19 vertices:



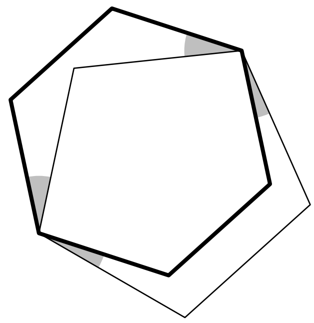
19 Here are some similar problems with polygons — in each, one can find the sum of the shaded angles.



a triangle
inside a square



a regular pentagon inside
a regular octagon

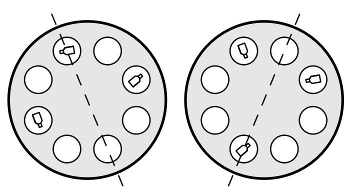


a regular hexagon
overlapping a regular hexagon

21 It would be interesting to get pupils to discuss and compare how they managed to solve this problem.

23 The largest six-digit palindrome that is a multiple of 7 is 999 999; what is the smallest? The largest seven-digit palindrome that is a multiple of seven is 9 994 999; what is the smallest?

25 There is a neater way to tackle this problem, once one realises that there are only two basic configurations, as shown in these two diagrams. In case (1), the three poppers are separated by two single gaps and one gap of three, whereas in case (2) there are two gaps of two and a single gap of one. For both configurations, the three poppers can be placed in eight distinct positions (by rotating successively by 45°). Because cases (1) and (2) cannot share a configuration, there are $2 \times 8 = 16$ distinct possibilities.



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